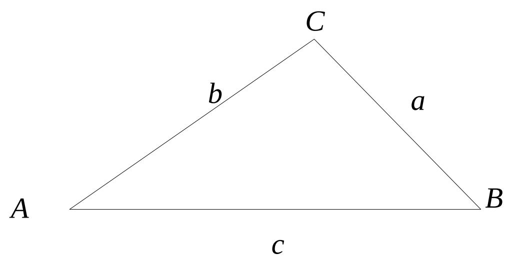
Solve Triangles (SAS): Solve for an Unknown Side

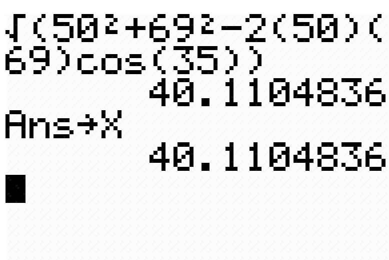
When you know two sides and an included angle of a triangle (SAS), you can use the Law of Cosines to solve for the other side. Consider the triangle ABC with the measures in the table.



$\angle A$=35º	$\angle B$=	$\angle C$=
a=	b=50	c=69

Since we know angle A we use $a^2 = b^2 + c^2 - 2bc\cos A$ from the Law of Cosines

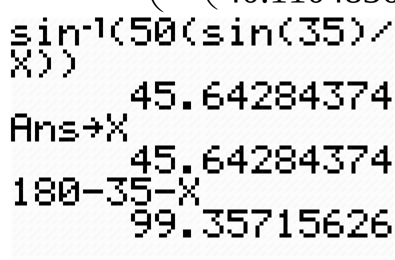
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$
$$a = \sqrt{50^2 + 69^2 - 2(50)(69)\cos 35}$$
$$a \approx 40.1104836$$



Note that we store that value in “x” for our next computation.

Now use the Law of Sines to solve for $\angle$B.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{50} = \frac{\sin 35}{40.1104836}$$
$$B = \sin^{-1}\left(50\left(\frac{\sin 35}{40.1104836}\right)\right) \approx 45.64^{\circ}$$

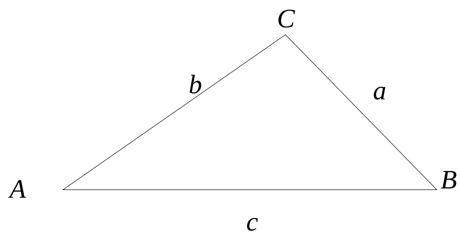


And $\angle C \approx 99.36^{\circ}$

$\angle A$=35º	$\angle B$≈45.64º	$\angle C$≈99.36º
a≈40.11	b=50	c=69

Solve Triangles (SSS): Solve for an Unknown Angle

When you know three sides of a triangle (SSS), you can use the Law of Cosines to solve for an angle. Consider the triangles ABC with the measures in the table.



$\angle A$=	$\angle B$=	$\angle C$=
a=40	b=50	c=70

We can solve for angle A using $a^2 = b^2 + c^2 - 2bc\cos A$

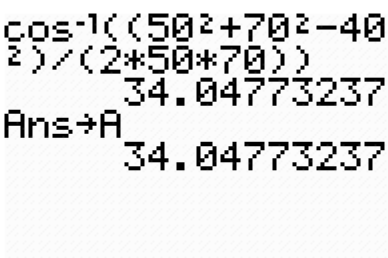
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\angle A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\angle A = \cos^{-1}\left(\frac{50^2 + 70^2 - 40^2}{2(50)(70)}\right)$$

$$\angle A \approx 34.05$$



Note that we store that value in "A" for a later computation.

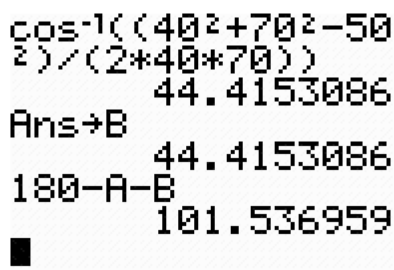
Now we could use the Law of Sines or Cosines to solve for $\angle B$. We will use $b^2 = a^2 + c^2 - 2ac\cos B$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\angle B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$\angle B = \cos^{-1}\left(\frac{40^2 + 70^2 - 50^2}{2(40)(70)}\right)$$

$$\angle B \approx 44.42$$



And $\angle C \approx 101.54^\circ$

$\angle A$≈34.05º	$\angle B$≈44.42º	$\angle C$≈101.54 º
a=40	b=50	c=70